LEARN: QUBIT - Advanced

A bit is the basic unit of classical information and is represented by a binary digit, indicated with 0 and 1. Its state is therefore either 0 or 1. For quantum systems, the special property of superposition allows for the state of the qubit to be in both states simultaneously, mathematically represented as a linear combination of the two corresponding basis vectors. Using the ket notation, the orthonormal basis vectors are usually identified as $|0\rangle$ and $|1\rangle$. Together, they form the so-called computational basis $\{|0\rangle, |1\rangle\}_{\mathcal{H}}$ for the qubit's Hilbert space. Therefore, the general state of a qubit can be written as

$$|\Psi\rangle = c_0|0\rangle + c_1|1\rangle\,,$$

where c_0 and c_1 are two complex numbers such that $|c_0|^2 + |c_1|^2 = 1$, due to the normalisation condition $\langle\Psi|\Psi\rangle = 1$.

While we can check whether a bit is in state 0 or 1, and this is in fact what classical computers do all the time, checking the state of a qubit requires performing a measurement, which generally alters its state (see Quest entry on measurement). A single measurement on a qubit only reveals partial information about its previous state. For instance, if we measure in the computational basis, we obtain the results 0 or 1 with probability $|c_0|^2$ and $|c_1|^2$, respectively. Despite their weirdness, the properties of qubit states described above are actually what lies at the core of quantum computation and quantum information. More precisely, quantum computing exploits quantum interference (see entries on superposition and wave-like behaviour) of many-qubit states to speed up calculations.

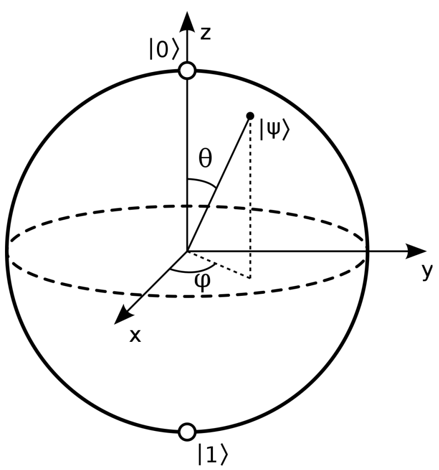
A useful way of visualising qubits is the following geometrical representation. Since $|c_0|^2 + |c_1|^2 = 1$, we can rewrite the state of the qubit as

$$|\Psi\rangle = e^{i\gamma}(\cos\tfrac{\theta}{2}|0\rangle + e^{i\varphi}\sin\tfrac{\theta}{2}|1\rangle)\,,$$

where γ, θ and φ are all real numbers. Notice that, while it may apparently seem that $|\Psi\rangle$ is identified by four degrees of freedom, as c_0 and c_1 are both complex numbers, the normalisation condition reduces this number to three. Moreover, since in Quantum Physics there are no observable effects due to global phases, we can ignore the factor $e^{i\gamma}$ and assume c_0 to be real, leading to

$$|\Psi\rangle = \cos\tfrac{\theta}{2}|0\rangle + e^{i\varphi}\sin\tfrac{\theta}{2}|1\rangle\,.$$

The numbers θ and φ can be identified as coordinates on a sphere (or, more precisely, a sphere with radius equal to 1), as shown in the figure.



This is the famous **Bloch sphere**, which can be exploited to visualise single-qubit states and often helps to illustrate ideas about quantum computation and information, since it can be easily used to represent graphically the operations that can be performed on single qubits.

A classical bit could only be at the North Pole or at the South Pole, where the states $|0\rangle$ and $|1\rangle$ are (notice, however, that this choice of the polar axis is completely arbitrary). A qubit state can be instead represented by any point on the surface, i.e., it can lie anywhere on the sphere.

In other words, the surface of the sphere represents the space of the qubit states, which is indeed a two-dimensional Hilbert space. Its two degrees of freedom can be then identified by the two angles θ and φ.