## LEARN: WAVE-LIKE BEHAVIOUR - Advanced

The physicists of the XIX century were used to identifying the state of a system with the outcome of a measurement. They did not consider the notion, introduced with Quantum Physics (QP), of the state of a system prior to the measurement being different to the one subsequent to it. Following Classical Mechanics and our entrenched habit of positioning and picturing objects in space, the founders of QP soon began to wonder *where* the quantum particles were placed, i.e., which is the result that we get when we measure their position. Let us therefore consider the position operator

$$\hat{X} = \int dx\, x\,|x\rangle\langle x| \qquad \text{with } x \text{ real numbers,}$$

where the kets $|x\rangle$ represent states[1] with a well-defined position $x$. According to the third postulate, the probability of obtaining a result $m_x$ related to the position $x$ when the particle is in an arbitrary state $|\Psi\rangle$ is given by

$$p^{X}_{|\Psi\rangle}(m_x) = |\langle x|\Psi\rangle|^2 .$$

Let's define $\Psi(x) \equiv \langle x|\Psi\rangle$, where $\Psi(x)$ is a complex-valued function; as it follows from the equation above, the probability for the result of the measurement to be $m_x$ is given by its square modulus $|\Psi(x)|^2$. The quantity $\Psi(x) = \langle x|\Psi\rangle$ is called *wave-function* of the system in the state $|\Psi\rangle$. Different interpretations of QP assign a different meaning to this object, and even if we will not review here such different perspectives, we find it important to clarify some points that often lead to confusion.

First of all, the wave-function is the position representation of the state of the system, from which we can calculate the associated probability of finding the system in a certain position. Bare in mind: $|\Psi(x)|^2$ does not represent the probability that the system *is* in the position $x$! The square modulus of the wave-function only provides the statistical distribution of the possible outcomes of many position measurements performed on many copies of the system in the same state. The particle, as a quantum system, can be in a superposition of states (see the Quest entry superposition) and that includes superposition of spatial locations $x$. The wave function is only another way of representing this mathematically. This is the origin of the experimentally observed interference (wave-like effects), e.g., in the famous double-slit experiment.

If you have read the third postulate of QP (see the Quest entry measurement), there's no surprise here. The wave-function concept introduced above follows naturally from the axiomatic structure of QP. In a similar way, we could consider $\langle p|\Psi\rangle$, the scalar product of the state of the system $|\Psi\rangle$ with the state $|p\rangle$ representing a state with a well-defined momentum $p$; it's an equivalent concept to $\langle x|\Psi\rangle$, but we don't call it wave-function. Perhaps the reason why we assign a special name to the position "representation" is due to the fact that we are used to imagining objects in space. This is why the "wave-function representation" of quantum particles became very popular and, to many, felt like a more familiar description. It's totally fine, but we have to keep in mind that it is only one of the possible representations of the state of a quantum object.

Some of you may have heard about the term "wave-particle duality". Let us clearly say it once and for all: the wave-particle duality, interpreted as *particles which are also waves, waves which are also particles*, does not exist. Strictly speaking, the sentence in italics above does not mean anything. An electron is a particle with a well-defined mass, and it doesn't have anything to do with waves propagating on a fluid, like those we observe when we look at the sea from the shore or when we throw a stone in a lake.

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[1] Note that the position states are not normalisable. Hence, strictly speaking, they are not physical states but just useful mathematical objects.

However, in some experiments, like the double-slit experiment, we observe quantum particles, like an electron, "interfering with itself". This does not mean that a single electron will give rise to interference fringes, as observed in waves. A single electron yields a point-like position outcome, but the probability distribution to find electrons at certain positions in space reveals a clear interference pattern. In this sense, it does make sense to talk about "wave-like behaviour" of quantum particles, but we have always to keep in mind that it's just an analogy.